



# Stage Separation Dynamic Analysis of Upper Stage of a Multistage Launch Vehicle using Retro Rockets

D. JEYAKUMAR AND K. K. BISWAS

Mission Design and Analysis Division, Launch Vehicle Design Entity  
Vikram Sarabhai Space Centre, Thiruvananthapuram - 695 022, India

B. NAGESWARA RAO\*

Structural Analysis and Testing Group, Aeronautics Entity  
Vikram Sarabhai Space Centre, Thiruvananthapuram - 695 022, India  
[bnrao52@rediffmail.com](mailto:bnrao52@rediffmail.com)

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**Abstract**—This paper presents in brief the formulation used for the studies of lateral and longitudinal dynamics associated with separation and pull out of an ongoing functional stage from the spent stage of a multistage launch vehicle using retro rockets as jettisoning system. A fourth-order Runge-Kutta method is employed to solve the governing nonlinear differential equations of motion to obtain the state vectors of the separating bodies. A statistical method is adapted to account for the off nominal design parameters of the bodies undergoing separation and the jettisoning system. The tolerances in magnitude of the pull out time, relative velocity, lateral gap and longitudinal clearances between the separating bodies were used to find out the nonoccurrence of collision between the separating bodies. This procedure helps in designing and adapting a suitable jettisoning system to achieve collision free separation of multistage launch vehicle stages. © 2005 Elsevier Ltd. All rights reserved.

**Keywords**—Jettisoning system, Retro-rocket, Jet damping, Pyro, Relative velocity, Relative distance, Launch vehicle.

## NOMENCLATURE

|            |                                           |                                                              |                                                                     |
|------------|-------------------------------------------|--------------------------------------------------------------|---------------------------------------------------------------------|
| $A_z$      | launch azimuth (deg)                      | $R_E, R_P, R_S$                                              | Earth's equatorial radius, polar radius, radius at the surface (km) |
| $h$        | altitude of separation (km)               | $r, q, p$                                                    | body angular velocity along yaw, pitch and roll axes (deg/s)        |
| $I$        | inertia tensor (kg-m <sup>2</sup> )       | $X, Y, Z$                                                    | body axes yaw, pitch and roll                                       |
| $t$        | Time (s)                                  | $\vec{F}_{\text{ext}} = (\vec{F}_g + \vec{F}_T + \vec{F}_J)$ | total external force due to gravity, thrust, and jet damping (N)    |
| $m_M, m_S$ | mass of the ongoing and spent bodies (kg) |                                                              |                                                                     |

\*Author to whom all correspondence should be addressed.

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|                                                              |                                                                    |                      |                                                                      |
|--------------------------------------------------------------|--------------------------------------------------------------------|----------------------|----------------------------------------------------------------------|
| $\vec{M}_{\text{ext}} = (\vec{M}_g + \vec{M}_T + \vec{M}_J)$ |                                                                    | $l_T$                | distance of thrust location from nose tip along longitudinal axis(m) |
|                                                              | total external moment due to gravity, thrust and jet damping (N-m) | $d_T$                | distance of thrust location from vehicle longitudinal axis (m)       |
| $\vec{r}_I$                                                  | position vector in ECI frame ( $X_I, Y_I, Z_I$ ) (m)               | $\psi, \theta, \phi$ | body orientation angles along yaw, pitch and roll in LPI frame (deg) |
| $\vec{r}_{CG}$                                               | position vector of the body CG offset (m)                          | $\Phi_{GC}$          | geocentric latitude of the launch station (deg)                      |
| $\vec{v}$                                                    | velocity in the body frame (m/s)                                   | $\Theta$             | launch station longitude (deg)                                       |
| $\dot{m}$                                                    | propellant mass flow rate (kg/s)                                   | $\mu$                | Earth's gravitational constant ( $m^3/s^2$ )                         |
| $T$                                                          | thrust (N)                                                         | $\Omega_E$           | Earth's angular velocity (rad/s)                                     |
| $r_T$                                                        | distance of thrust location from main body CG (m)                  |                      |                                                                      |
| $l_{CG}$                                                     | distance of CG from nose tip along longitudinal axis (m)           |                      |                                                                      |

## 1. INTRODUCTION

Launch vehicles almost always employ multistage using the concept of staging for boosting payload capabilities. This in turn requires jettisoning the spent stages. The dynamics of separating bodies has received the attention of several investigators [1–24]. Chubb [2] has constructed collision boundaries between two separating stages. Dwork [3] and Wilke [4] provide valuable insight into disturbances caused by separation mechanisms in a spinning setup. Considerable amount of aerodynamic data including stability derivatives have been generated for space shuttle type of configurations involving winged bodies [5,6]. The data so gathered have been utilized for separation dynamics investigation [8], Waterfall [9] investigated multispring system for separation and spinning and nonspinning bodies. Longren [10] analyzed spin stabilized rockets with guide shoes and rails constraining the lateral motion. Hurley and Carrie [11] reviewed the genesis of a four bar linkage separation system and carried out the analysis for separation of parallel staged shuttle vehicles. Subramanyam [13] developed a general model for spring assisted stage separation. Biwas [15] investigated several aspects of separation for strap-on from the launch vehicle. The aerodynamics of separation is rather complex due to relatively large angles of incidence, intricate geometry, and the involved interference phenomenon. Moraes Jr. *et al.* [16] have developed the Navier-stoke's code, which provided a good agreement with the experiments only on the nose part of the ongoing stage. Lochan *et al.* [17–20] have analyzed the separation dynamics of strap-on boosters from the core rocket utilizing the wind tunnel simulation data for dynamic forces. Cheng [21] has developed an analytical procedure based on a coupled gas / structure model to simulate the fairing separation events. The procedure was validated by comparing the analysis results with full-scale payload fairing separation test data of Titan IV launch vehicle. Seongjin Choi *et al.* [22] have developed an efficient three-dimensional aerodynamic-dynamic coupled code to simulate the separation dynamics of strap-on boosters in the dense atmosphere. Jeyakumar and Biswas [23–25] have provided stage separation system design and dynamic analysis of ISRO launch vehicles.

A typical launch vehicle may involve several separation events such as strap-on separation, stage separation, heat shield separation, ullage rocket separation and spacecraft separation (see Figure 1). In a multistage rocket configuration the most important event is staging. The staging process during the flight of a launch vehicle comprises of a series of events, which takes place in transferring the regime of an ongoing stage to that of the next stage. Obviously, the process commences from the detection of the burn out of the ongoing stage, and continues with the ignition of the next stage and the separation of the spent stage. Ignition of the next stage and the separation of the spent stage may takes place concurrently or with a time difference and in any order, depending on vehicle configuration and mission requirements, and they may be

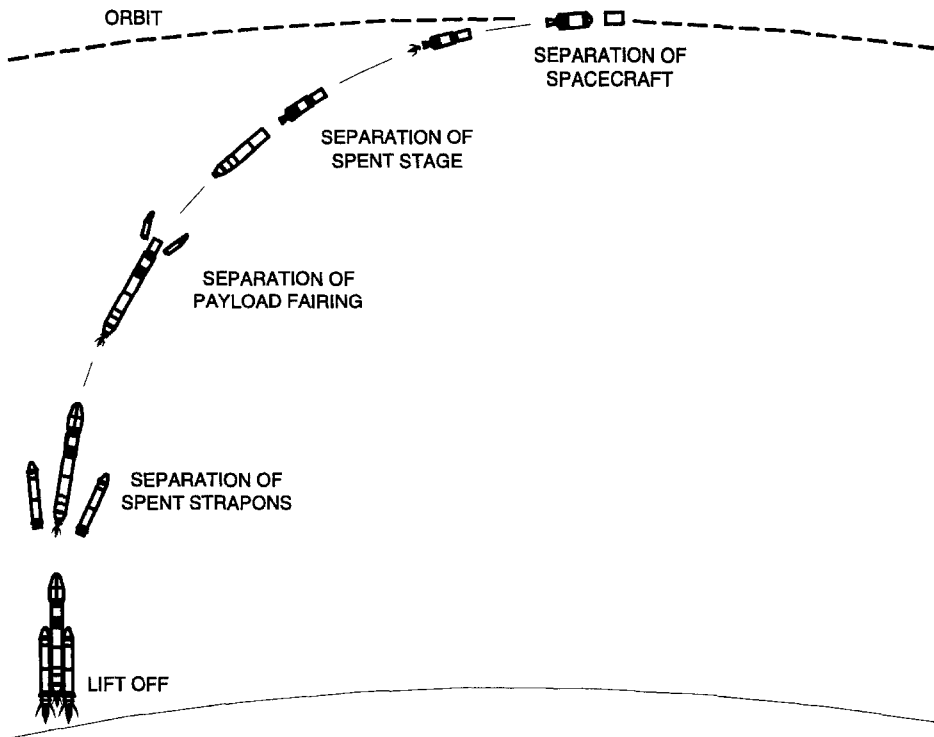


Figure 1. Separation events of a typical launch vehicle.

programmed before lift off or on the basis of real time in flight monitoring. During the staging process, the orientation control of the vehicle has to be maintained within specified limits - to be defined on consideration of controllability, structural integrity and avoidance of recontact. Successful staging depends on the proper functioning of the various vehicle systems under the environments and disturbances acting on the vehicle. Separation involves pyro-actuated severance of the designated vehicle elements and imparting of the required relative velocity between the separating bodies through passive or active means. The pyro actuator has to have the capacity with specified minimum margin to do the severance and yet it should not generate shock levels strong enough to derange the modules of the vehicle. The modules located in the vicinity of the severance zone are susceptible to the severance shock. The relative velocity between the separating bodies is generated either by releasing a set of precompressed springs or by firing auxiliary rockets. In this activity, angular motion is also unavoidably imparted to the separating bodies.

For carrying higher payloads, the launch vehicles are often configured to use solid strapon motors. When strapon motors are employed one major concern is the differential thrust among the motors. The differential thrust is highest during the tail off period, which is also the zone of stage separation. This places high demands on control power plant and hence is to be limited. The problems gets complicated when the control power available is also small in nonautonomous control systems like the secondary injection or flex nozzle control systems commonly employed in solid motors. During the separation event, the vehicle control has to pass from the separated stage to the succeeding stage. Where both the stages are solid motors and control is through injection of a fluid into the nozzle divergence (secondary injection thrust vector control (SITVC)), some special problems arise, during stage separation. In these cases, control is available only during the motor operation. Even within motor operation, SITVC effectiveness diminishes for motor chamber pressures below a certain value. This situation occurs during the later portion of motor tail-off and during the initial phase of ignition. Hence, if the succeeding stage is not ignited before the pressure drops below a certain value in the earlier stage, loss of vehicle control occurs.

This will result in building up of vehicle orientation errors. This error build-up in the presence of atmospheric dynamic pressure loads on the vehicle could prove detrimental to the vehicle and the mission. A major factor to be considered in designing the separation system and flight sequence is the tail off thrust. Consider the case of separation between two core stages, with lower stage being a solid motor. During the lower stage operation, the vehicle control is obtained by the stage motor action itself (by SITVC or flex nozzle control). The stage separation should be achieved and the upper stage should take over control without creating large error in orientation. The separation should be carried out before the pressure in the booster solid motor (lower stage) drops to such a low value that its control is not effective. But if separation is done too early, the thrust from the booster stage (tail off thrust) leads to collision between the two separated stages. To ensure collision free separation, retro rockets are incorporated on the separated booster. The sizing of the retro rockets (thrust levels), booster tail off thrust, dynamic pressure on vehicle, vehicle configuration at interstage, separation system force should all be analyzed interactively in the separation system design. When the solid motor is followed by liquid engine there is a requirement of certain level of positive acceleration before the liquid engine ignition. The ullage motor is provided to ensure this level at separation. The ullage motor is ignited before the thrust from the lower stage solid motor drops below a certain level.

This paper presents a mathematical formulation for the analysis of stage separation using retro motors as jettisoning systems in a multistage launch vehicle. The problem is modeled under the influence of forces and moments of the individual bodies undergoing separation, treating the bodies as rigid. A rigid body has six degrees of freedom (three displacement of a point fixed in the body and three orientation angles). Each body undergoing separation has twelve state variables (namely three displacement of point fixed in the body, three orientation angles, three components of velocity vector, and three components of angular velocity vector). Two basic types of frames of reference are used in the formulation apart from the usual geo-centric or Earth centered inertial (ECI) frame of reference (to compute gravity effect) and the topo-centric or launch point inertial (LPI) frame. The first is a local inertial frame (LI-frame) in which the dynamics is described and is essentially the body frame of the combined body rotationally frozen at the instant of separation command but free to travel at a velocity equal to the combined vehicle velocity at separation. This enhances the computation accuracy and makes the results of the analysis easy to comprehend. The second type of frame is the body coordinate system (B-frame), separately for each of the body undergoing separation. The transformation from body frame to local inertial frame and vice versa can be achieved through a transformation matrix in which a prefixed sequence of rotation of the Euler angles is used.

In principle, equations of motion can be expressed in any coordinate system. It is advantageous to express them in the body coordinate system, as the moment of inertia, engine tail-off thrust and aerodynamic computations are very simple in the body coordinate system while gravity computation is simpler in geo-centric inertial coordinate system. Transformation from one coordinate frame to another is made in the present study as the forces and moments are defined in different coordinate systems. The nonlinear ordinary differential equations of motion are solved for a typical retro-rocket assisted separation system using a fourth-order Runge-Kutta integration scheme. For separation dynamic analysis, it is necessary to simulate the trajectories of the separating bodies taking into account of the forces and moments acting on the bodies. To understand the clearances between the separating bodies, critical points will be identified on these bodies. The relative distance between the critical points, relative velocity, body angular velocity and body orientation angles will be estimated for the identification of collision hazards during separation. The trace of the points together with body orientation angles will confirm collision free clean separation and build up of any tip of errors on the ongoing functional stage. Graphical simulation of separating bodies will show the occurrence of collision (if any) during separation.

## 2. FORMULATION

The physical problem is to simulate the dynamic characteristics of the separating rigid bodies under the influence of forces and moments acting on them. Each individual body has twelve state variables (namely three displacement of point fixed in the body, three orientation angles, three components of velocity vector, and three components of angular velocity vector). In principle, equations of motion can be expressed in any coordinate system. It is preferable to express them in the body coordinate system, as the moment of inertia, engine tail off thrust, and aerodynamic computations are very simple in the body coordinate system. Figures 2 and 3 show the different coordinate frames used in the present study. The  $X_I$ ,  $Y_I$ , and  $Z_I$  axes as shown in the Figure 2 lies in the equatorial plane of the earth. Gravitational force terms are presented in this coordinate system. The  $X_T$ ,  $Y_T$ , and  $Z_T$  axes define the Topocentric or Launch point Inertial (LPI) coordinate system. The ECI and LPI coordinates are related through the launch azimuth ( $A_z$ ), launch site geo-centric latitude ( $\Phi_{GC}$ ) and launch site longitude ( $\Theta$ ). The vehicle orientation at any time is defined with the Euler angles, *viz.*, yaw orientation angle ( $\psi$ ), pitch orientation angle ( $\theta$ ), and roll orientation angle ( $\phi$ ) with respect to LPI frame. The body frame coincides with LPI frame at the time of launch. It has its origin at the center of gravity (CG) of the individual bodies, namely main (ongoing) body and separating (spent) body as shown in the Figure 3. The  $X_M$ ,  $Y_M$ , and  $Z_M$  axes with  $M_{CG}$  as origin define the main body coordinate system. The  $X_S$ ,  $Y_S$ , and  $Z_S$  axes with  $S_{CG}$  as origin define the separating body coordinate system. The  $X_L$ ,  $Y_L$ , and  $Z_L$  axes coincide with body axes of the combined body just before separation. This frame moves with a constant velocity same as that of the combined body at the instant of separation.

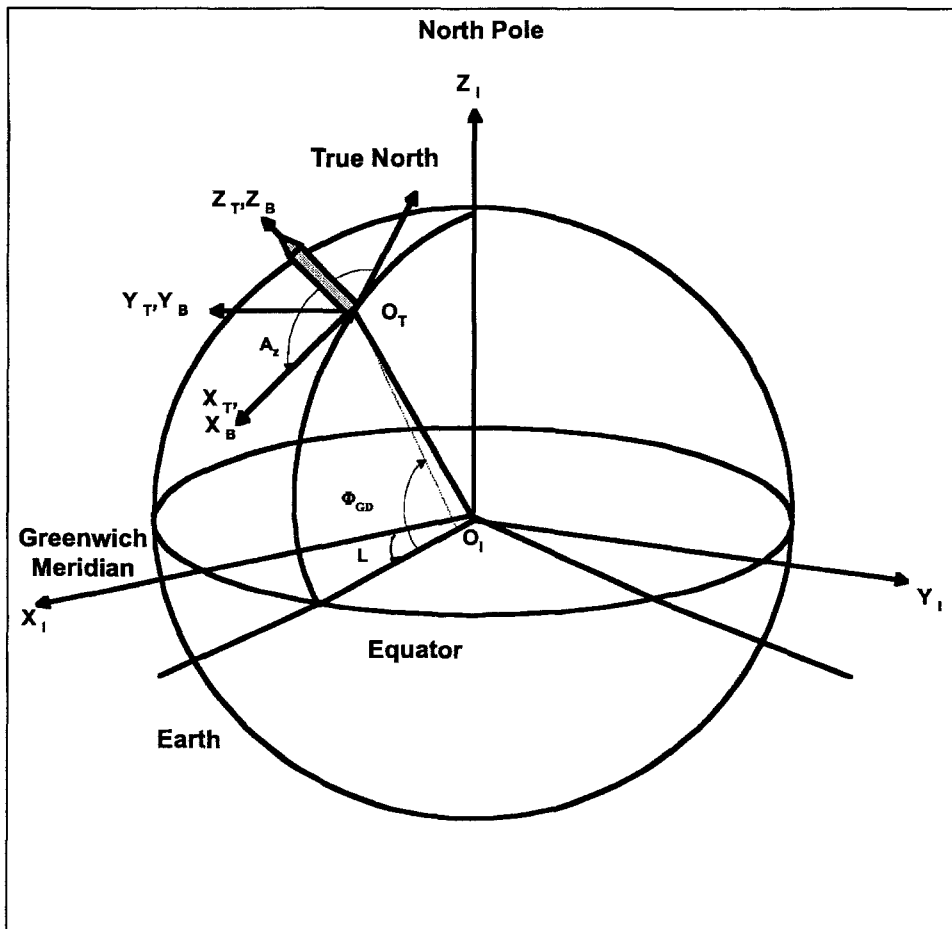


Figure 2. Earth centered inertial (ECI) and launch point inertial (LPI) frames.

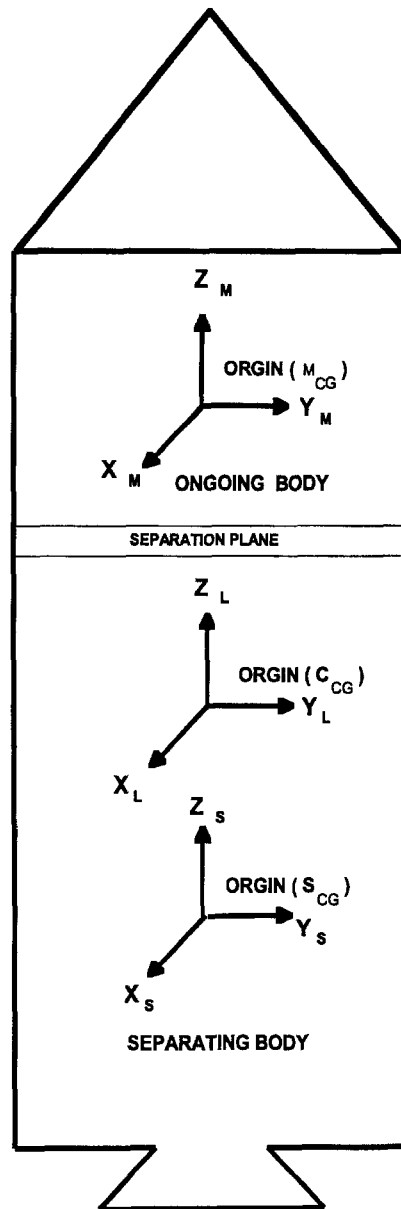


Figure 3. Body coordinate systems.

## 2.1. Equations of Motion

The translational and rotational equations of motion for each body in the respective body frame are [7,23–29]

$$m \left\{ \frac{d\vec{v}}{dt} \right\} + \vec{\omega} \times \vec{v} = \vec{F}_{\text{ext}}, \quad (1)$$

$$\frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L} = \vec{M}_{\text{ext}}. \quad (2)$$

Here  $m$  is the mass of the body;  $\vec{v} = v_x \vec{i}_b + v_y \vec{j}_b + v_z \vec{k}_b$ , is the velocity;  $\vec{\omega} = r \vec{i}_b + q \vec{j}_b + p \vec{k}_b$ , is the angular velocity, which has components  $r$ ,  $q$ , and  $p$  along the body axes, viz., yaw, pitch and roll respectively;  $t$  is the time;  $\vec{F}_{\text{ext}} = F_x \vec{i}_b + F_y \vec{j}_b + F_z \vec{k}_b$ , is the total external force vector;  $\vec{L} = L_x \vec{i}_b + L_y \vec{j}_b + L_z \vec{k}_b$ , is the angular momentum vector;  $\vec{M}_{\text{ext}} = M_x \vec{i}_b + M_y \vec{j}_b + M_z \vec{k}_b$ , is the total external moment vector.  $\vec{i}_b, \vec{j}_b, \vec{k}_b$  are the unit vectors with respect to the center of mass of the respective body frame.

The components of angular momentum vector are obtained from

$$\begin{Bmatrix} L_x \\ L_y \\ L_z \end{Bmatrix} = [I] \begin{Bmatrix} r \\ q \\ p \end{Bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{Bmatrix} r \\ q \\ p \end{Bmatrix} \quad (3)$$

Here  $I_{xx}$ ,  $I_{yy}$ , and  $I_{zz}$  are the principal moment of inertia about yaw, pitch and roll (X, Y, Z) axes, respectively.  $I_{xy}$ ,  $I_{xz}$ ,  $I_{yx}$ ,  $I_{yz}$ ,  $I_{zx}$ , and  $I_{zy}$  are the products of inertia with respect to the indicated planes, e.g.,  $xy$ ,  $xz$ ,  $yz$ . The symmetry in the products of inertia has been noted. It may be noted that the moments of inertia may only be positive, but the products of inertia may be either positive or negative. The  $3 \times 3$ -inertia matrix on the right of equation (3) is called the inertia dyadic [7,10].

Equations (1) and (2) can be written in the form

$$\frac{d}{dt} \begin{Bmatrix} v_x \\ v_y \\ v_z \end{Bmatrix} = \begin{Bmatrix} pv_y - qv_z \\ rv_z - pv_x \\ qv_x - rv_y \end{Bmatrix} + \frac{1}{m} \begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix}, \quad (4)$$

$$\frac{d}{dt} \begin{Bmatrix} r \\ q \\ p \end{Bmatrix} = [I]^{-1} \begin{Bmatrix} pL_y - qL_z + M_x \\ rL_z - pL_x + M_y \\ qL_x - rL_y + M_z \end{Bmatrix}. \quad (5)$$

The kinematic differential equations, which provide the relationship between Euler angles and angular velocity components, are

$$\frac{d}{dt} \begin{Bmatrix} \psi \\ \theta \\ \phi \end{Bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ \cos \psi & \cos \psi & 0 \\ \sin \phi \frac{\sin \psi}{\cos \psi} & \cos \phi \frac{\sin \psi}{\cos \psi} & 1 \end{bmatrix} \begin{Bmatrix} r \\ q \\ p \end{Bmatrix}. \quad (6)$$

The motion of bodies in yaw plane is important in separation dynamics. Hence, the sequence  $(\psi, \theta, \phi)$  corresponding to the yaw, pitch, roll is followed in the present study, whereas the sequence for simulation of flight trajectories adopted is  $(\theta, \psi, \phi)$ . There are singularities in two of the rates of the Euler angles  $\theta$  and  $\phi$ , which occur in both pitch and roll when  $\psi = 90^\circ$ . These singularities in equation (6) are eliminated using L'Hospital's rule [27, pp. 160–163] and one can obtain expressions that are finite when  $\psi = 90^\circ$ . In other ways one can resort to Euler four parameters approach and enforce a constraint on the parameters with Lagrange multipliers. For vertically launched vehicles the yaw angles ( $\psi$ ) will be well below  $90^\circ$ .

Equation (4) to (6) are the governing equations of motion for the individual bodies undergoing separation. These are nonlinear ordinary differential equations, which are to be solved numerically after specifying the components of total external force and moment vectors. The dynamic characteristics of the separating bodies are examined here under the influence of the gravity and the spring forces and moments.

## 2.2. ECI to LPI Transformation

The transformation matrix  $[T_{EL}]$ , which transfers the components of a vector from ECI frame to LPI frame, is [25–27]

$$[T_{EL}] = [e_{ij}]_{3 \times 3}. \quad (7)$$

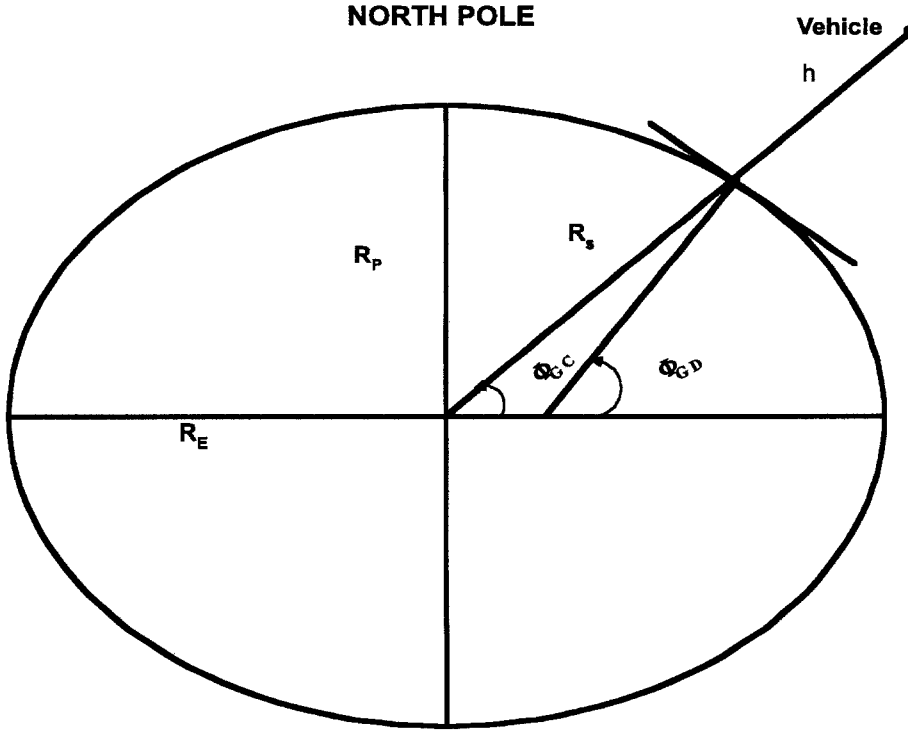


Figure 4. Oblate earth geometry.

Where

$$\begin{aligned} e_{11} &= -\cos A_z \sin \Phi_{GC} \cos \Theta - \sin A_z \sin \Theta; & e_{12} &= -\cos A_z \sin \Phi_{GC} \sin \Theta + \sin A_z \cos \Theta; \\ e_{13} &= \cos A_z \cos \Phi_{GC}; \end{aligned}$$

$$\begin{aligned} e_{21} &= -\sin A_z \sin \Phi_{GC} \cos \Theta + \cos A_z \sin \Theta; & e_{22} &= -\sin A_z \sin \Phi_{GC} \sin \Theta - \cos A_z \cos \Theta; \\ e_{23} &= \sin A_z \cos \Phi_{GC}; \end{aligned}$$

$$e_{31} = \cos \Phi_{GC} \cos \Theta; \quad e_{32} = \cos \Phi_{GC} \sin \Theta; \quad e_{33} = \sin \Phi_{GC};$$

The geocentric latitude of the vehicle position is,  $\Phi_{GC} = \sin^{-1}(Z_I/r_I)$ ; the geodetic latitude of the vehicle position is,  $\Phi_{GD} = \tan^{-1}\{(R_E/R_P)^2 \tan \Phi_{GC}\}$ ;  $R_P$  is the polar radius of the oblate earth geometry; the inertial longitude of the vehicle position is  $\Theta_I = \tan^{-1}(Y_I/X_I)$ ; the relative longitude with respect to earth is,  $\Theta_R = \Theta_I - \Omega_E t$ ; and the earth rotation rate,  $\Omega_E = 7.29211 \times 10^{-5}$  rad/s. The altitude of the vehicle in Figure 4 is  $h = r_I - R_S$ ; the radius of the earth corresponding to  $\Phi_{GC}$  is  $R_S = R_E / \sqrt{1 + \{(R_E/R_P)^2 - 1\} \sin^2 \Phi_{GC}}$ ; the equatorial radius  $R_E = 6378.1656$  km; and the polar radius,  $R_P = 6356.7838$  km. The launch station longitude of  $\Theta$  80.23621 deg, geodetic launch latitude of  $(\Phi_{GD})$  13.73204 deg and launch azimuth of  $(A_z)$  102 deg E are used in the current studies.

### 2.3. LPI to Body Transformation

The transformation matrix  $[T_{LB}]$ , which transfers the components of a vector from LPI frame to body frame, is [25–27]

$$[T_{LB}] = [b_{ij}]_{3 \times 3}, \quad (8)$$



Where

$$\begin{aligned}
 b_{11} &= \cos \phi_0 \cos \theta_0 + \sin \phi_0 \sin \psi_0 \sin \theta_0; & b_{12} &= \sin \phi_0 \cos \psi_0; \\
 b_{13} &= -\cos \phi_0 \sin \theta_0 + \sin \phi_0 \sin \psi_0 \cos \theta_0; \\
 b_{21} &= -\sin \phi_0 \cos \theta_0 + \cos \phi_0 \sin \psi_0 \sin \theta_0; & b_{22} &= \cos \phi_0 \cos \psi_0; \\
 b_{23} &= \sin \phi_0 \sin \theta_0 + \cos \phi_0 \sin \psi_0 \cos \theta_0; \\
 b_{31} &= \cos \psi_0 \sin \theta_0; & b_{32} &= -\sin \psi_0; \\
 b_{33} &= \cos \psi_0 \cos \theta_0.
 \end{aligned}$$

Here  $\psi_0$ ,  $\theta_0$ ,  $\phi_0$  are the initial Euler angles. It should be noted that the transformation matrix,  $T_{LB}$ , which transfers the components of a vector from the local inertial frame (LI) to body frame, is nothing but the transformation matrix  $T_{LB}$  having the elements as a function of the sequence of rotation with Euler angles  $\psi$ ,  $\theta$ , and  $\phi$ .

#### 2.4. Gravity Force and Moment Computation

The gravitational acceleration along the ECI frame is computed as [27,28]

$$\vec{g}_{EC} = \vec{g}_{XI}\vec{e}_x + \vec{g}_{YI}\vec{e}_y + \vec{g}_{ZI}\vec{e}_z = -\text{grad } U. \quad (9)$$

Where the gravitational potential,

$$U = -\frac{\mu}{r_I} \left[ 1 - \frac{1}{2}J_2\eta^2(3\zeta^2 - 1) - \frac{1}{2}J_3\eta^3(5\zeta^3 - 3\zeta) - \frac{1}{8}J_4\eta^4(35\zeta^4 - 30\zeta^2 + 3) \right], \quad (10)$$

$\eta = R_E/r_I$ ;  $\zeta = Z_I/r_I$ ;  $\vec{r}_I = X_I\vec{e}_x + Y_I\vec{e}_y + Z_I\vec{e}_z$ , is the vehicle position vector with respect to the ECI frame;  $r_I = \sqrt{\vec{r}_I \cdot \vec{r}_I}$ , is the radial distance of the vehicle from the earth center;  $R_E$  is the equatorial radius of the oblate earth geometry (see Figure 4);  $\vec{e}_x$ ,  $\vec{e}_y$ ,  $\vec{e}_z$  are the unit vectors along the axes of the ECI frame; The gravitational constant,  $\mu = 3.9860253 \times 10^{14} \text{ m}^3/\text{s}^2$ ; gravitational harmonics  $J_2 = 1.0827 \times 10^{-3}$ ;  $J_3 = -2.3 \times 10^{-6}$ ;  $J_4 = -1.8 \times 10^{-6}$ . Since the gravitational acceleration (9) is computed in ECI frame, it has to be transformed to the LPI frame. The gravitational acceleration along the body frame,  $\vec{g} = g_X\vec{i}_b + g_Y\vec{j}_b + g_Z\vec{k}_b$  computed using the transformation matrices  $[T_{EL}]$  and  $[T_{LB}]$ . The components of the gravity forces in the body frame are

$$\begin{Bmatrix} g_X \\ g_Y \\ g_Z \end{Bmatrix} = [T_{LB}][T_{EL}] \begin{Bmatrix} g_{XI} \\ g_{YI} \\ g_{ZI} \end{Bmatrix} \quad (11)$$

The gravity force and moment are computed as

$$\vec{F}_g = m\vec{g}, \quad (12)$$

$$\vec{M}_g = \vec{r}_{CG} \times \vec{F}_g. \quad (13)$$

Where  $\vec{r}_{CG} = X_{CG}\vec{i}_b + Y_{CG}\vec{j}_b + Z_{CG}\vec{k}_b$ , is the position vector of the vehicle center of mass. In general, the center of gravity does not coincide with the center of mass of the vehicle. The gravitational forces will cause moment about the center of mass of the vehicle. However, since vehicle dimensions are very small compared to the distance from the earth center, the moment due to gravitational forces is small compared to other moments.

## 2.5. Retro Rocket Thrust and Moment Computation

The motion of a spent stage separating from its functional rocket stage by the force of retro thrust is considered. The separation impulse is provided by release of the energy stored in the relatively low thrust short burn duration solid motor called retro motors. Its location and support hardware design is dictated by the geometry requirements, as the jet impingement from the retro motor should not affect the highly sensitive payload interface. For each of these thrusts following information are required. Time measured from initiation of separation versus thrust in tabular form. Thrust action point with respect to CG of separating body on which thrust is acting. Mounting azimuth angle with respect to vehicle axis. Nozzle exit area, if separation is in the atmosphere and the thrust given is vacuum thrust then the correction is made as follows: [29] magnitude of actual thrust = vacuum thrust – (atmospheric pressure \* nozzle exit area).

To compute the actual thrust from a propulsion unit in body frame, a transformation matrix is generated between the thrust frame (coordinate frame assumed at the nozzle) and the body frame as shown in Figure 5). This matrix is defined by the following sequence of anti clockwise rotation of the body frame: about Z-axis through  $\rho$  (measured from Y-axis) the azimuth angle defining the propulsion unit location; about X-axis through  $\lambda$ - nozzle cant angle as shown in Figure 6.

The thrust vector in the body frame is:

$$\vec{F}_T = T(t) \left( \sin \lambda \sin \rho \vec{i} - \sin \lambda \cos \rho \vec{j} + \cos \lambda \vec{k} \right). \quad (14)$$

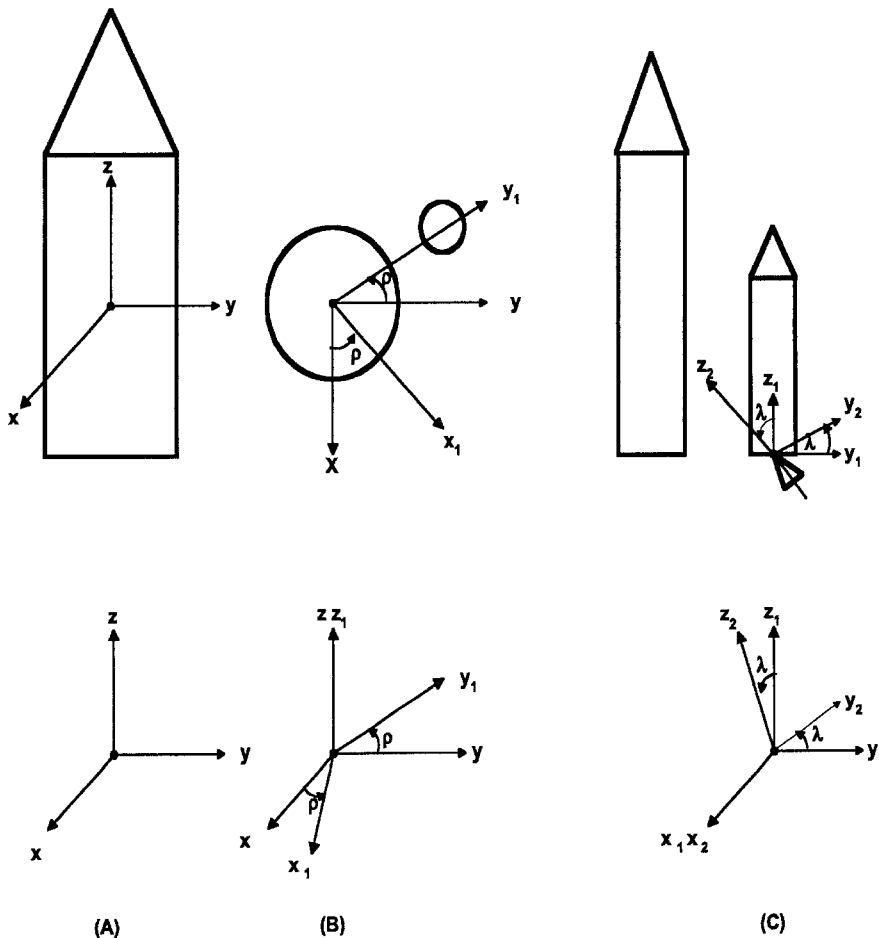


Figure 5. Transformation of retro-rocket thrust frame to body frame.

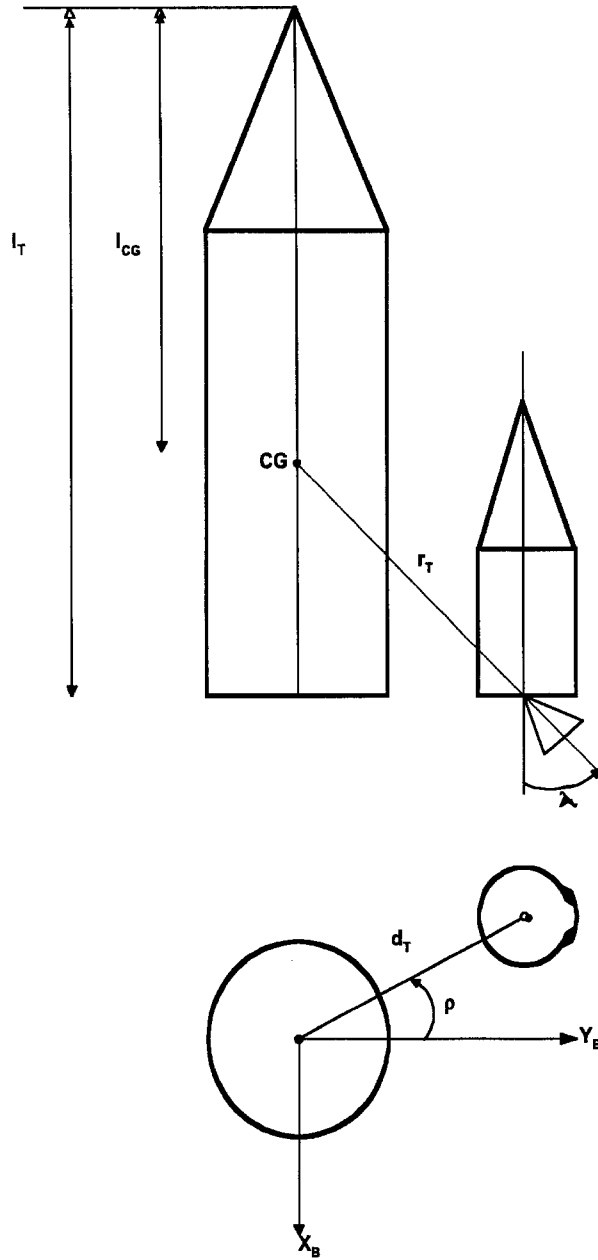


Figure 6. Thrust acting in a retro rocket with respect to main body.

Propulsion unit thrust location is given by:

$$\vec{r}_T = d_T \sin \rho \vec{i} + d_T \cos \rho \vec{j} + (l_{CG} - l_T) \vec{k}. \quad (15)$$

Moment due to thrust is given by

$$\vec{M}_T = \vec{r}_T \times \vec{F}_T. \quad (16)$$

## 2.6. Jet Damping Force and Moments

Considering a rotating vehicle having an initial pitch rate, and with its motor burning, reveals that the mass of gas ejected from the nozzle will have angular momentum. Since the mass is isolated from the vehicle once it leaves the nozzle, it is clear that the total angular momentum of the vehicle itself must decrease. Consequently, angular momentum is no longer conserved

within the vehicle and due to this fact alone; the gyroscopic motion of the vehicle is lessened (jet damping effect). The jet damping force and moment are given by [7,29]

$$\vec{F}_J = -\dot{m}\vec{\omega} \times \vec{r}_T, \quad (17)$$

$$\vec{M}_J = \vec{r}_T \times \vec{F}_J, \quad (18)$$

where  $-\dot{m}$  is the mass flow rate per unit area (considered a negative quantity since it decreases the vehicle mass). These jet damping forces and moments are computed in the body coordinate system.

## 2.7. Net Force and Moments

Using equations (12) to (18), the total external force and moment vectors in equations of motion (1) and (2) are written as

$$\vec{F}_{\text{ext}} = \vec{F}_g + \vec{F}_T + \vec{F}_J, \quad (19)$$

$$\vec{M}_{\text{ext}} = \vec{M}_g + \vec{M}_T + \vec{M}_J. \quad (20)$$

With these specified forces and moments, the solution of equation (4) to (6) will give the dynamic characteristics of the individual bodies undergoing separation.

## 2.8. Identification of Collision between Separating Bodies

For separation dynamic analysis, it is necessary to simulate the trajectories of the separating bodies taking into account of the forces and moments acting on the bodies. To understand the clearances between the separating bodies, critical points will be identified on these bodies. The relative distance between the critical points, relative velocity, body angular velocity and body orientation angles will be estimated for the identification of collision hazards during separation. The trace of the points together with body orientation angles will confirm collision free clean separation and build up of any tip of errors on the ongoing functional stage. Graphical simulation of separating bodies will show the occurrence of collision (if any) during separation [23–25].

After determining the velocities of the individual bodies at a time ‘ $t$ ’, the relative velocity between the separating bodies in the body frame is calculated. Applying the inverse transformation to the relative velocity and integrating with respect to time, the actual separation distance between the bodies is found. The integration of the equations of motion is continued till the retro rocket is thrusting. To account the variations in the dynamic parameters, a statistical analysis is adopted.

## 3. STATISTICAL VARIATION OF A FUNCTION

In launch vehicle separation system design process, it is with the combination of many variable parameters. For this purpose a multivariate concept, which involves more variable parameters, will be more useful [30].

Let the variable function be denoted as  $\Psi$ . The variable  $\Psi$  be functionally related to ‘ $n$ ’ random variables, namely,  $x_1, x_2, \dots, x_n$ . This function is assumed to be continuous and differentiable. It can be expressed into a Taylor series about a particular point say  $x_1^*, x_2^*, \dots, x_n^*$ . Neglecting terms of higher order and assuming that the random variables are independent with means at  $x_1^*, x_2^*, \dots, x_n^*$ , respectively, the expected (average) value of the function  $\Psi$  is approximately given by

$$E(\Psi) \approx \Psi(x_1^*, x_2^*, x_3^*, \dots, x_n^*) + \frac{1}{2} \sum_{i=1}^n \text{Var}(x_i) \frac{\partial^2 \Psi}{\partial x_i^2}, \quad (21)$$

and the variance of the function,  $\Psi$  is approximately given by

$$\text{Var}(\Psi) \approx \sum_{i=1}^n \text{Var}(x_i) \left( \frac{\partial \Psi}{\partial x_i} \right)^2. \quad (22)$$

The partial derivatives of  $\Psi$  in equations (18) and (19) are taken at the point  $x_1^*, x_2^*, \dots, x_n^*$ , and thus, are constants. Further more, if the  $x_i$  are normally distributed,  $\Psi$  is approximately normally distributed [30].

If the tolerances of the individual variables all fall within their specification limit, then the tolerances for the function,  $\Psi$  with 99.9% confidence level will not exceed  $E(\Psi) \pm 3\sqrt{\text{var}(\Psi)}$ . The square root of the variance of a quantity is the standard deviation of that quantity.

In the present study, there are 14 variable functions namely velocity components ( $v_x, v_y, v_z$ ), position components ( $x, y, z$ ), angular velocity ( $r, q, p$ ), body orientation components ( $\psi, \theta, \phi$ ), pull out time and lateral gap available at the critical point (nozzle end) during pull out. These variable functions are related to the dynamic parameters of the separating bodies like mass, moment of inertia, CG offset and separation rocket parameters like cant angle, thrust, ignition delay, etc. For the variations in the specified dynamic parameters, the above described statistical analysis will be useful to find the expected value and variance of the 14 variable functions for the ongoing and spent body. With these values, body rate for ongoing and spent body, relative velocity and relative distance between the separating bodies can be estimated. Finally it is possible to estimate the tolerances of the physical quantities.

For the present problem, the random variables  $x_1, x_2, \dots, x_n$  are considered as the specified dynamic parameters. The variable functions  $\Psi_i$  ( $i = 1$  to 14) correspond to velocity components ( $v_x, v_y, v_z$ ), position components ( $x, y, z$ ), angular velocity components ( $r, q, p$ ), and the body orientation angles ( $\psi, \theta, \phi$ ), pull out time and lateral gap available at the critical point (nozzle end) during pull out. The values of  $\Psi_i$  are obtained for the average values of dynamic parameters. To find the values of first and second derivatives of  $\Psi_i$  with respect to the dynamic parameters the following procedure is adopted.

For each variable function  $\Psi_i$ , the functional values generated by specifying a value of dynamic parameter close to its average and the rest of the dynamic parameters are specified by their average values. The generated data is fitted in to a cubic polynomial of the varied dynamic parameter. The first and second derivative of the function with respect to that dynamic parameter is found out from the cubic polynomial at its average value. This procedure is repeated for all the variable functions and dynamic parameters to obtain the first and second derivative of variable functions with respect to each dynamic parameter at its average value. Considering variances in the dynamic parameters and the first and second derivatives of variable functions with respect to those dynamic parameters equations (21) and (22) give the expected value of the functional and its variance.

#### 4. NUMERICAL RESULTS

Studies are made on the lateral and longitudinal dynamics associated with separation and pull out of an ongoing functional stage from the spent stage of a multistage launch vehicle using retro rockets as jettisoning systems. Figure 7 shows the location of retro motors placed in pair's diametrically opposite and the specified pull out length to be cleared to achieve the collision free separation. The specified pull out length is 4.6 m. Figure 8 shows the magnitude of actual thrust-time curve for the retro motors. Tables 1 and 2 gives the trajectory and dynamic parameters of reference [23] considered for the separating bodies at the time of separation. Table 3 provides the dispersion parameters considered in the studies for the propulsion units. A fourth-order Runge-Kutta integration scheme is used to solve the nonlinear ordinary differential equations with a fixed time step size of 0.001 s. Table 4 provides the expected value and the variance of the

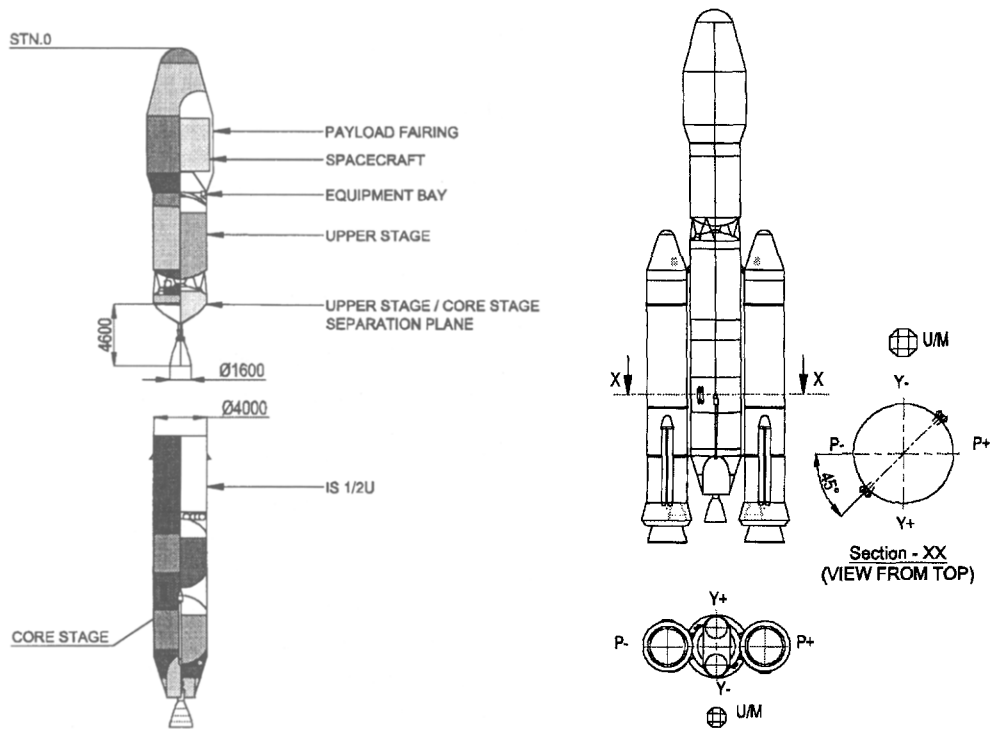


Figure 7. Pull out geometry of the separating bodies and the location of retro rockets in a typical launch vehicle.

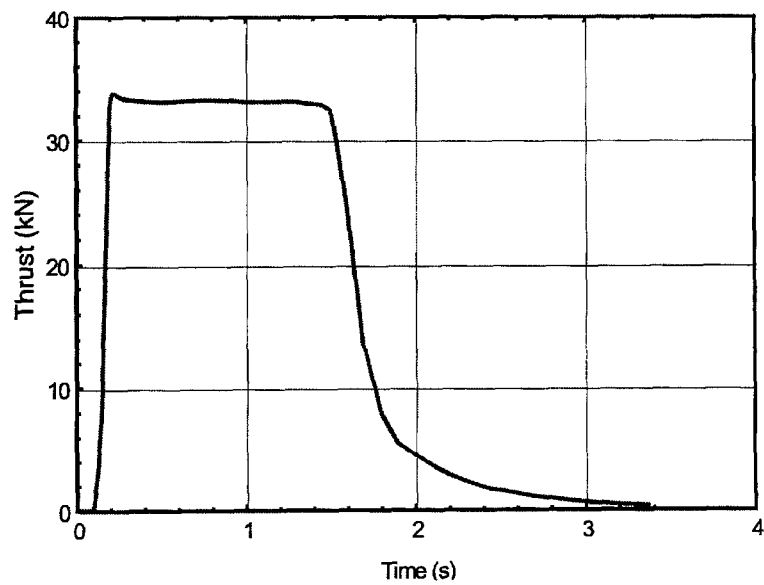


Figure 8. Thrust versus time of retro rockets.

Table 1. Trajectory parameters at the time of separation.

|                                                      |                                           |
|------------------------------------------------------|-------------------------------------------|
| Time of separation ' <i>t</i> ' (s)                  | 300                                       |
| Altitude of separation ' <i>h</i> ' (km)             | 140                                       |
| ECI Position $X_I, Y_I, Z_I$ (m)                     | 460290.075, 6354251.31, 1432314.34        |
| Initial body rate $r, q, p$ (deg/s)                  | 2 for ongoing body and 2.1 for spent body |
| Initial orientation $\Psi_0, \theta_0, \phi_0$ (deg) | 0.0, 74.5, 0.0                            |

Table 2. Dynamic parameters of the separating bodies.

| Parameters                    | Ongoing Body      | Spent Body        |
|-------------------------------|-------------------|-------------------|
| Mass (kg)                     | $34000 \pm 100$   | $12000 \pm 100$   |
| Z (mm)                        | $31000 \pm 100$   | $16000 \pm 100$   |
| Y (mm)                        | $30 \pm 10$       | $50 \pm 10$       |
| X (mm)                        | $30 \pm 10$       | $50 \pm 10$       |
| $I_{ZZ}$ (kg-m <sup>2</sup> ) | $32000 \pm 10\%$  | $50000 \pm 10\%$  |
| $I_{YY}$ (kg-m <sup>2</sup> ) | $400000 \pm 10\%$ | $700000 \pm 10\%$ |
| $I_{XX}$ (kg-m <sup>2</sup> ) | $400000 \pm 10\%$ | $700000 \pm 10\%$ |
| $I_{XY}$ (kg-m <sup>2</sup> ) | $-50 \pm 100\%$   | $-150 \pm 100\%$  |
| $I_{XZ}$ (kg-m <sup>2</sup> ) | $-50 \pm 100\%$   | $-150 \pm 100\%$  |
| $I_{YX}$ (kg-m <sup>2</sup> ) | $-50 \pm 100\%$   | $-150 \pm 100\%$  |
| $I_{YZ}$ (kg-m <sup>2</sup> ) | $-50 \pm 100\%$   | $-150 \pm 100\%$  |
| $I_{ZY}$ (kg-m <sup>2</sup> ) | $-50 \pm 100\%$   | $-150 \pm 100\%$  |
| $I_{ZX}$ (kg-m <sup>2</sup> ) | $-50 \pm 100\%$   | $-150 \pm 100\%$  |

Table 3. Sensitivities of the propulsion units.

| Parameters                                       | Dispersion level |
|--------------------------------------------------|------------------|
| Jettisoning rocket thrust dispersion (%)         | $\pm 3.5$        |
| Jettisoning rocket ignition delay (ms)           | $100 \pm 20$     |
| Jettisoning rocket cant angle (o)                | $\pm 0.3$        |
| Differential thrust between the core engines (s) | $\pm 5$ kN       |
| Thrust line offset between the core engines (mm) | $\pm 0.3$        |

Table 4. Variations in 12 degrees of freedom for the separating bodies.

| Physical Quantities | Ongoing Body     |                |                  | Separated Body   |                |                  |
|---------------------|------------------|----------------|------------------|------------------|----------------|------------------|
|                     | $-3\sigma$ Value | Expected Value | $+3\sigma$ Value | $-3\sigma$ Value | Expected Value | $+3\sigma$ Value |
| $v_x$ (m/s)         | 10.53984         | 10.84993       | 11.16001         | 9.77148          | 10.15609       | 10.54070         |
| $v_y$ (m/s)         | -2.69797         | -2.62983       | -2.56169         | -2.33249         | -1.92549       | -1.51848         |
| $v_z$ (m/s)         | -4.05700         | -3.93076       | -3.80452         | -13.02184        | -12.88692      | -12.75201        |
| $x$ (m)             | 6.52351          | 6.92109        | 7.31868          | 5.93653          | 6.31467        | 6.69281          |
| $y$ (m)             | -1.87954         | -1.78350       | -1.68746         | -1.31949         | -1.1757        | -1.03244         |
| $z$ (m)             | 5.25069          | 5.39504        | 5.53940          | -14.95139        | -14.78274      | -14.61409        |
| $r$ (deg/s)         | 1.41327          | 1.41340        | 1.41353          | -0.06321         | 1.51740        | 3.09800          |
| $q$ (deg/s)         | 1.41616          | 1.41631        | 1.41646          | 0.83178          | 1.51716        | 2.20254          |
| $p$ (deg/s)         | 0.59143          | 0.59272        | 0.594            | 0.55548          | 0.60112        | 0.64676          |
| $\psi$ (deg)        | 1.70384          | 1.75528        | 1.80673          | 0.80480          | 1.88540        | 2.96600          |
| $\theta$ (deg)      | 1.70256          | 1.75409        | 1.80561          | 1.38802          | 1.87811        | 2.36820          |
| $\phi$ (deg)        | 0.68658          | 0.70813        | 0.72969          | 0.67446          | 0.71442        | 0.75439          |

12 degrees of freedom of the separated bodies. Table 5 provides the tolerances of the parameters of the separating bodies with 99.9% confidence level. It is noted from Table 5 that the expected pull out time is 1.24 s and its tolerance is from 1.2 s to 1.276 s. The initial longitudinal distance just before pull out between the CG of the separating bodies is 15.561 m. The expected relative longitudinal distance between the CG of the individual body at the specified separation time is 20.19588 m; hence, the actual pull out completed at the specified separation time is 4.63488 m.

The magnitude of the body rate is:

$$\sqrt{r^2 + q^2 + p^2}. \quad (23)$$

Table 5. Tolerances of the parameters of the separating bodies with 99.9% confidence level.

| Parameters                         | $-3\sigma$ Value | Expected Value | $+3\sigma$ Value |
|------------------------------------|------------------|----------------|------------------|
| Pull out time (s)                  | 1.20416          | 1.24056        | 1.27696          |
| Relative velocity (m/s)            | 8.81871          | 9.00902        | 9.19933          |
| Relative longitudinal distance (m) | 19.97342         | 20.19588       | 20.41835         |
| Lateral gap available (mm)         | 646.70596        | 801.44444      | 956.18293        |
| Body rate for ongoing body (o/s)   | 2.08646          | 2.08685        | 2.08724          |
| Body rate for the spent body (o/s) | 1.09493          | 2.26818        | 3.44127          |

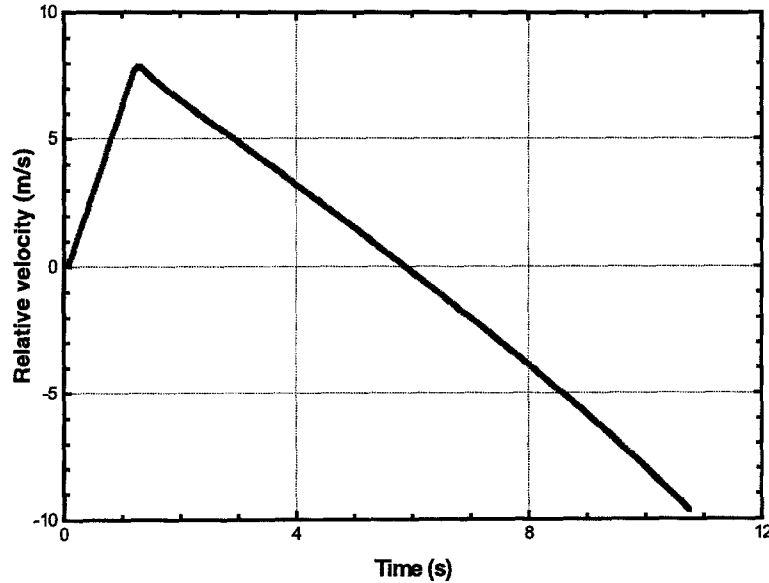


Figure 9. Relative velocity of the separating bodies with time.

The expected body rate of the ongoing stage is found to be 2.08685 deg/s, whereas in the case of the spent stage, it is found to be 2.26818 deg/s.

The relative velocity ( $v_R$ ) of the separating bodies is:

$$v_R = \sqrt{(v_{x1} - v_{x2})^2 + (v_{y1} - v_{y2})^2 + (v_{z1} - v_{z2})^2}. \quad (24)$$

The relative distance ( $d_R$ ) from the CGs of the separating bodies is:

$$d_R = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}. \quad (25)$$

Here  $(v_{x1}, v_{y1}, v_{z1})$ ,  $(v_{x2}, v_{y2}, v_{z2})$ ,  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  are the components of the velocity vector and position vector of the ongoing and spent body which were transformed to common body inertial (CBI) frame. Figures 9 and 10 show the variation of the relative velocity and relative longitudinal distance between the separated bodies with separation time. It can be seen from Figure 9 that the relative velocity gradually increases upto 8 m/s till the retro action time. The average velocity is found to be 4 m/s. It can be seen from Figure 10 that the relative longitudinal distance increases with separation time up to 6 sec due to retro action time and decreases with time due to the spent stage tail-off thrust. Hence to avoid collision between the separating bodies, the ongoing functional stage should be ignited sufficiently before 10 sec. The stage sequencing should be made accordingly. Figure 11 shows the animation of the stage pull out at the beginning of separation and Figure 12 shows the animation of the separated bodies at the end of separation. In the present analysis nozzle end diameter is 1.6 m. The spent body diameter is 4 m; the clearance available (excluding the attachments) for the pull out is 1 m. From



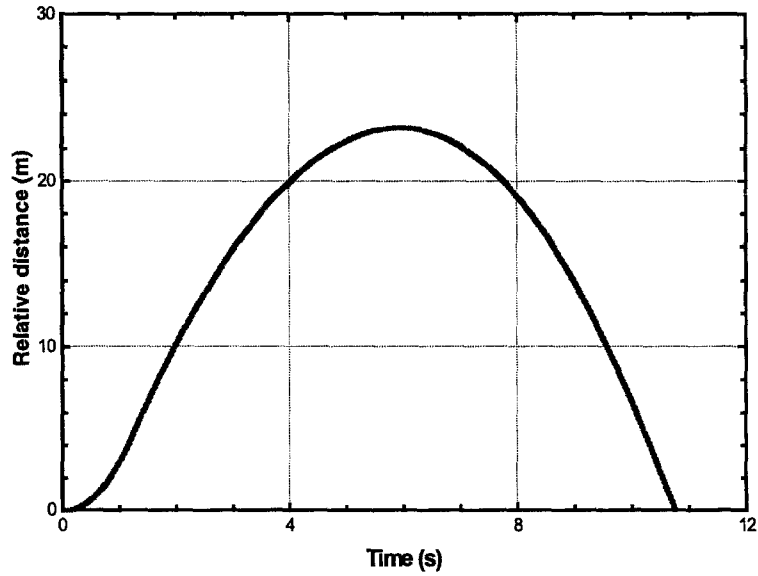


Figure 10. Relative longitudinal distance between the separating bodies with time.

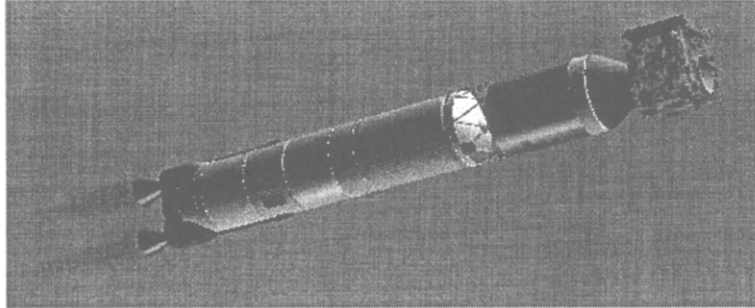


Figure 11. Animation of the stage at the starting of separation.

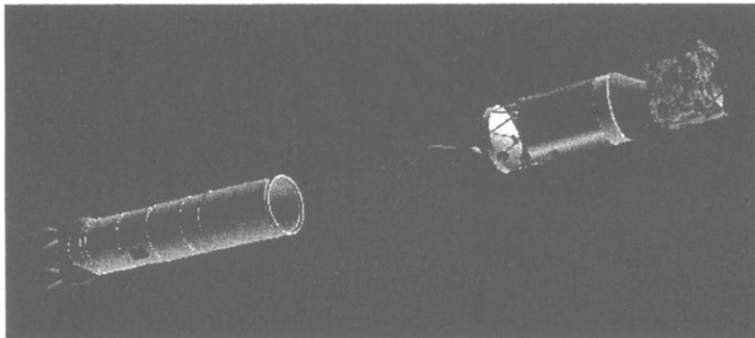


Figure 12. Animation of the stage pull out at the end of separation.

this study, it is noted that the lateral gap consumed during the nozzle pull out of 4.6 m is 200 mm due to the variations in the dynamic parameters of the separating bodies and the jettisoning systems. It is noted from the results in Table 5 that the expected lateral gap between the nozzle end and the burn out body is 800 mm. Hence a collision free separation of the bodies can be expected.

## 5. CONCLUSIONS

Dynamic studies are made on the stage separation process and pull out of ongoing stage nozzle from the spent stage of a multistage rocket carrier using retro rocket motors. A statistical method

is followed to examine the influence of the design variables on the separating bodies. It is noted that the lateral gap consumed is about 200 mm against the available gap of 1000 mm during the pull out of 4.6 m ongoing stage nozzle from the spent stage, which ensures a collision free separation.

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